The dynamics of hyperfine coupling in Ag in an arbitrarily oriented magnetic field

M. Brieger

Institute of Technical Physics, DLR (German Aerospace Center)
Pfaffenwaldring 38-40, D-70569, Stuttgart, Germany
Tel ++49-711-6862-516, Fax ++49-711-6862-349
E-mail: Michael.Brieger@dlr.de

In this contribution, we investigate by rigorously solving the time-dependent Schrödinger equation the full dynamics of hyperfine (hf) coupling in the presence of an arbitrarily oriented magnetic field as encountered in a Stern-Gerlach experiment [1]. The simplest conceivable system that nevertheless has experimental relevance as it applies to the stable isotopes of silver, is the coupling of an electronic ground state spin $J = 1/2$ with a nuclear spin $I = 1/2$. Despite its seeming simplicity, this problem already pushes the endeavor to the verge of analytical feasibility as it calls for the solution of a four-dimensional system of coupled linear differential equations of first order. But the final benefit from this effort of analytically looking in full rigor into what is happening to the atoms in a Stern-Gerlach experiment, lies in the following findings: to each Ag atom leaving the orifice of the oven which is assumed to be in a field-free location, has to be assigned an individual axis of quantization that complies with the actual orientation of its total angular momentum. This is the axis relative to which the eigenstates under hf interaction are defined for the individual atom. However, in the overwhelming number of cases, it will not coincide with the direction of the field. Would only the direction matter, there would be no problem as a rotational transformation would express an unchanged physical situation relative to the new direction by a linear superposition of the original hf eigenfunctions $|\gamma\,;1/2,1/2\,;F,M_F\ >$. The field, however, is exerting torques on the magnetic moments related to the coupling angular momenta. These not only effect an orientational change of the total angular momentum but also change its magnitude due to changing the coupling angle. The latter applies especially to the original orientation $M_F = 0$. As a result, the tip of the vector resulting from the expectation values for the components of the magnetic moment, performs a complicated three-dimensional periodic motion in the field that not only manifests an orientational change but also a simultaneous change of magnitude and so expresses a field-imposed nutation that reminds of the cycloid motion of classical cases. Depending on the relative orientation with respect to the local field direction, there usually is a time-averaged stationary component of the magnetic moment that allows an inhomogeneous field to exert a force in the direction of the field gradient. As a result, silver atoms that started in the oven in one of the two *stretched* hf eigenstates $|\gamma\,;1/2,1/2\,;F,M_F = \pm 1 >$ and so usually are not in a stationary state of the field when flying through an inhomogeneous part of it, there suffer a deflection due to a force that is exerted by the field gradient and given by, respectively, $\pm\,\frac{1}{2}(g_J\mu_B + g_I\mu_N)\cos\Theta\nabla B$ where Θ is the polar angle between the field direction and the axis of quantization of the individual atom when leaving the oven. This expression is identical to the classical one. But only atoms which happen to have $\Theta = 0$, are in stationary states with respect to the field. On the other hand, atoms leaving the oven in states represented by linear combinations with respect to their individual axes of quantization, $\alpha \,|\, \gamma\,; 1/2, 1/2\,; F = 1,\, M_F = 0 > \exp{(-i\,\omega_1\, t)} + \beta \,|\, \gamma\,; 1/2, 1/2\,; F = 0,\, M_F = 0 > \exp{(-i\,\omega_o\, t)}$, experience the force $-\frac{1}{2}(g_J \mu_B - g_I \mu_N) \cos\Theta(|A_{0-1/2}|^2 - |A_{0+1/2}|^2)\nabla B$ where $A_{0\pm1/2}$ are the initial amplitudes of the uncoupled basis states $|\,\gamma\,; J = 1/2, M_J = 0 \mp 1/2\,; I = 1/2, M_I = \pm 1/2 >$. In the whole spectrum of values that comply with the condition of normalization $|A_{0-1/2}|^2 + |A_{0+1/2}|^2 = 1$ only the combinations $A_{0-1/2} = 1/\sqrt{2}$ and $A_{0+1/2} = \pm 1/\sqrt{2}$ lead to above hf eigenstates for $M_F = 0$, respectively. In a field-free environment, these special cases describe situations of dynamical balance where for the special coupling angles related to the Clebsch-Gordan coefficients the torques mutually exerted by the coupling angular momenta, cancel each other and so provide the torque-free condition that is necessary for any angular momentum's being stationary (cf the 2nd contribution to this conference). The result shows that if a natural occupation were only allowed for the original hf eigenstates, 50 % of the atoms would pass the inhomogeneous field undeflected, contrary to the experimental result. But if we assume what can be easily justified by the many vigorous collisions occuring in the 1323K hot oven [1] before a silver atom can leave it and, additionally, the weakness of the hf coupling, namely, that the occupation of the $M_F = 0$ level is equally open to all conceivable nonstationary *precession states with nutation* [2], i.e., all allowed combinations of the amplitudes $A_{0\pm1/2}$, we obtain a similar, though flatter deflection pattern than for the *stretched* states.

Concluding we can state that exploiting the full scope of solutions that the principle of superposition is offering leads to quasi-classical results.

[1] W. Gerlach and O. Stern, *Ann. Physik* **74** 673 (1924).

[2] M. Brieger and H. A. Schuessler, *Europhys. Lett.* **34** 319 (1996); *ibid.* **35** 1 (1996).